Enrollment No: ______________________ Exam Seat No: ______________________

C. U. SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Topology

Subject Code: 4SC06TOP1 **Branch: B.Sc. (Mathematics)**

Semester: 6 **Date: 06/05/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **[14]**

a) Define: Topology **(02)**

b) Define: Closure of a set **(01)**

c) True/False: If A is closed set then $A^\circ = A$. **(01)**

d) Give an example of set which is open as well as closed. **(02)**

e) If $a \in R$ then $\{a\}$ is closed set in R. **(02)**

f) Give an example of $\overline{A \cap B} \neq \bar{A} \cap \bar{B}$. **(02)**

g) If X=R and $A = Q$ then find A'. **(01)**

h) Is every indiscrete topological space X is connected? **(01)**

i) State Hereditary property **(01)**

j) True/False: Every finite set is compact. **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions **[14]**

a) Define lower limit topology and prove it. **(05)**

b) Define Door space and give an example of **(05)**
i) (X, τ) is a door space and ii) (Y, τ) is not a door space.

c) Let $X = \{1,2,3,4\}, \tau = \{\phi, X, \{1\}, \{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$. If $A = \{1,3\}$ then find $A^\circ, \bar{A}$. And also give an example of set which is not closed. **(04)**

Q-3 Attempt all questions **[14]**

a) Let (X, τ) be a topological space and A be a subset of X then prove that $\bar{A} = A \cup A'$. **(05)**

b) Prove that $\tau_C = \{U \mid X - U \text{ is countable or } X - U = X\}$ is topology on set X. **(05)**

c) Let (X, τ) be a topological space and A,B are two subsets of X then prove that $(A \cap B)^\circ = A^\circ \cap B^\circ$. Also verify $(A \cup B)^\circ \neq A^\circ \cup B^\circ$ with example. **(04)**

Q-4	**Attempt all questions**		**[14]**
a)	Let (X, τ) be a topological space and Y be a non-empty subset of X then prove that the collection $\tau_y = \{U \cap Y / U \in \tau\}$ is a topology on Y.		**(06)**
b)	Let (X, τ) be a topological space and A,B be two subsets of X then $(A \cup B)' = A' \cup B'$.		**(04)**
c)	Define: Exterior point and find it for $A = (0,1)$ of R.		**(04)**
Q-5	**Attempt all questions**		**[14]**
a)	Let Y be a subspace of X then prove that a set A is closed in Y iff$A = Y \cap C$, where C is closed in X.		**(06)**
b)	Let (X, τ) be a topological space and A,B be two subsets of X then $\overline{A \cup B} = \bar{A} \cap \bar{B}$.		**(04)**
c)	Let (X, τ) be a topological space and A,B be two subsets of X then prove that i) $Bd\ A = \bar{A} \cap \overline{X - A}$ and ii) $Bd\ (A \cap B) \subset Bd\ A\ \cup Bd\ B$.		**(04)**
Q-6	**Attempt all questions**		**[14]**
a)	Let X and Y be topological space and $f: X \to Y$ then following are equivalent: i) f is continuous ii) for every subset A of X then $f(\bar{A}) \subset \overline{f(A)}$ iii) for evey close set B in Y then $f^{-1}(B)$ is closed in X.		**(07)**
b)	Let X be a topological space. A and B be two open subsets of X then A and B are separated$A \cap B = \phi$.		**(04)**
c)	Define Continuous function with an example.		**(03)**
Q-7	**Attempt all questions**		**[14]**
a)	Prove that Homeomorphism is an equivalence relation in the collection of topological spaces.		**(07)**
b)	Define Disconnected space with an example.		**(04)**
c)	Define Compact set with an example.		**(03)**
Q-8	**Attempt all questions**		**[14]**
a)	Let X be a topological space then X is disconnected iff there exist a non-empty proper subset of X which is both open and closed in X.		**(07)**
b)	Define: T_1 −space and Give an example of topological space which is T_1 −space but not T_2 −space.		**(07)**